

COMPLETE PARALLAX AND PROPER-MOTION SOLUTIONS FOR HALO BINARY-LENS MICROLENSING EVENTS

ANDREW GOULD¹ AND NIKOLAY ANDRONOV

Ohio State University, Department of Astronomy, Columbus, OH 43210; gould@astronomy.ohio-state.edu, andronov@astronomy.ohio-state.edu

Received 1998 September 29; accepted 1998 December 8

ABSTRACT

A major problem in the interpretation of microlensing events is that the only measured quantity, the Einstein timescale t_E , is a degenerate combination of the three quantities one would like to know: the mass, distance, and speed of the lens. This degeneracy can be partly broken by measuring either a “parallax” or a “proper motion,” and can be completely broken by measuring both. Proper motions can easily be measured for caustic-crossing binary-lens events. Here we examine the possibility (first discussed by Hardy & Walker) that one could also measure a parallax for some of these events by comparing the light curves of the caustic crossing as seen from two observatories on Earth. We derive analytic expressions for the signal-to-noise ratio of the parallax measurement in terms of the characteristics of the source and the geometry of the event. For Galactic halo binary lenses seen toward the LMC, the light curve is delayed from one continent to another by a seemingly minuscule 15 s (compared to $t_E \sim 40$ days). However, this is sufficient to cause a difference in magnification on the order of 10%. To actually extract complete parallax information (as opposed to merely detecting the effect) requires observations from three noncollinear observatories. Parallaxes cannot be measured for binary lenses in the LMC, but they can be measured for Galactic halo binary lenses seen toward M31. Robust measurements are possible for disk binary lenses seen toward the Galactic bulge, but are difficult for bulge binary lenses.

Subject headings: dark matter — Galaxy: halo — gravitational lensing — Magellanic Clouds

1. INTRODUCTION

One of the major problems in the interpretation of microlensing observations is that for most events the only physically relevant parameter extracted from the light curve is the Einstein timescale, t_E , which is a complicated combination of three quantities that one would like to know individually: the mass of the lens, the distance to the lens, and the transverse speed of the lens relative to the observer-source line of sight. For example, more than 12 candidate events have been found toward the Large Magellanic Cloud (LMC; Alcock et al. 1997b; Aubourg et al. 1993), but it is still not known if these are predominantly due to a new population of objects that comprise half or more of the mass of the halo, or if they are a previously unrecognized stellar structure either in the LMC itself (Sahu 1994; Wu 1994) or along the line of sight toward the LMC (Zhao 1998; Zaritsky & Lin 1997; Evans et al. 1998). Similarly, several hundred events have been discovered toward the Galactic bulge (Udalski et al. 1994; Alcock et al. 1997a), and these could potentially be very useful to address questions of Galactic structure (Zhao, Rich, & Spergel 1996) and the stellar mass function (Zhao, Spergel, & Rich 1995; Han & Gould 1996; Gould 1996a). However, the threefold degeneracy among mass, distance, and speed makes such an analysis extremely difficult and subject to distortions from unknown systematic effects.

A number of ideas have been advanced to partially or totally break this threefold degeneracy. Gould (1992) showed that for sufficiently long events ($t_E \gtrsim \gamma r/2\pi$), the reflex motion of Earth induces a distortion of the light curve that yields a “parallax,” essentially the (two components of the) transverse velocity of the lens projected onto the plane of the observer. Several parallaxes have now been measured

toward the bulge (Alcock et al. 1995; Bennett et al. 1997), and important information has been extracted from the lack of a parallax detection in a long event seen toward the Small Magellanic Cloud (SMC; Palanque-Delabrouille et al. 1998; Afonso et al. 1998). Parallaxes cannot ordinarily be measured for short events in this way because Earth does not change its velocity enough during the course of the event to produce a sufficient distortion of the light curve. However, even for short events one can sometimes gain some information from the “parallax asymmetries” induced in the light curve (Gould, Miralda-Escudé, & Bahcall 1994), and this information, while substantially less useful than a full parallax, could nevertheless be important in some applications (Buchalter & Kamionkowski 1997; Gould 1998).

Parallaxes could be routinely measured for a large fraction of events by launching a satellite into solar orbit (Refsdal 1966; Gould 1994b, 1995a; Boutreux & Gould 1996; Gaudi & Gould 1997; Marković 1998). Because the Einstein ring is usually of order of a few AU, the light curve of the event is substantially different as seen from Earth and the satellite. This enables one to determine essentially the time it takes for the event to get from one to the other, and therefore (since the Earth-satellite distance is known) determine the projected transverse velocity. Unfortunately, no dedicated parallax satellite is currently planned. However, it will be possible to combine observations by the *Space Infrared Telescope Facility* with intensive ground-based observation to obtain parallaxes for some events (Gould 1999).

Hardy & Walker (1995) showed that it is possible to obtain some parallax information for binary-lens events by comparing the light curves of the caustic crossing as seen from two observatories on Earth. The shorter baseline (relative to a satellite) is compensated by the rapid change in the magnification. They also showed that by comparing

¹ Alfred P. Sloan Foundation Fellow.

observations from three noncollinear observatories, one could measure the full parallax.

A complementary type of additional information can be obtained from measurements of the proper motion of the lens relative to the observer-source line of sight. Because t_E is known, measuring the proper motion, μ , is equivalent to measuring the angular size of the Einstein ring, $\theta_E = \mu t_E$. Numerous ideas have been advanced to measure this quantity. If a single lens transits the face of the source, the light curve will deviate from the point-source approximation, which yields the source transit time, and therefore (since the angular size of the source is approximately known) the proper motion (Gould 1994a; Nemiroff & Wickramasinghe 1994; Witt & Mao 1994). The probability of such transits is low because the source size is much smaller than θ_E . However, Alcock et al. (1997c) have measured this effect for one bulge lensing event, and several other measurements have also been made but not yet published. Transits are extremely rare toward the LMC and SMC, in part because the source angular radii are smaller and in part because there are many fewer events. To date no single-lens transit events have been observed toward the LMC or SMC. On the other hand, for binary-lens events, the proper motion can be determined whenever the source crosses the caustic (region of formally infinite magnification) by dividing the source size by the measured caustic-crossing time. Caustic-crossing binary-lens events comprise of order 5% of all microlensing events, so this is a potentially very effective method. Measurement of the crossing time requires much more detailed coverage of the light curve than is available from the roughly nightly observations used to find microlensing events. However, three groups now intensively monitor ongoing events, GMAN (Alcock et al. 1997c), PLANET (Albrow et al. 1998), and MPS (Rhie et al. 1999). In particular, one of the only two events seen toward the SMC was a binary lens, and the measurement of its proper motion demonstrated that it is almost certainly in the SMC itself and not a halo lens (Afonso et al. 1998; Albrow et al. 1999a; Alcock et al. 1999; Udalski et al. 1998; Rhie et al. 1999). There are several other suggestions for measuring proper motions, including a spectroscopic method (Maoz & Gould 1994) and a binary-source method (Han & Gould 1997).

If both the parallax and the proper motion were measured, there would then be three measured quantities (including t_E) and three unknowns (mass, distance, and speed), so a complete solution would be possible (Gould 1992, 1995b). To date, four practical ideas have been devised to obtain complete solutions, and each is applicable to only a relative handful of events.

First, if a dedicated parallax satellite were launched, then parallaxes would be measured for most events, and the relatively small fraction for which proper motions could be obtained would then have complete solutions. Unfortunately, as mentioned above, no such mission is currently planned.

Second, the proposed *Space Interferometry Mission* (SIM) could measure both the proper motion and the parallax of some events (Boden, Shao, & Van Buren 1998). During microlensing events, the centroid of the two images is typically deflected by several tens of microarcseconds relative to the source position. The pattern of deviation is an ellipse whose size yields θ_E . Since SIM has an astrometric accuracy of $4 \mu\text{as}$, it can measure this deviation quite

well. The reflex motion of Earth produces an additional deviation that is superimposed on this ellipse, and by measuring this deviation one can determine the parallax. However, very few lensing events toward the Magellanic Clouds have sources brighter than $V = 20$ (the SIM limit). Near the magnitude limit, very long integration times are required to reach the nominal precision of $4 \mu\text{as}$, which implies that it will be possible to obtain complete solutions for only a handful of events. Toward the bulge, the situation is much more favorable because there are many events with bright ($V \lesssim 16$) sources. See Gould & Salim (1999).

Third, complete solutions are possible for a significant fraction of extreme microlensing events (EMEs; Gould 1997). By definition, EMEs have peak magnifications $A_{\text{max}} \gtrsim 200$. This high magnification permits measurement of the parallax by observing the event from two different locations from Earth. Recall that the reason for launching a parallax satellite was to get an observatory far enough away (in units of the Einstein ring) so that the event would appear significantly different. Two continents on Earth are separated by only $\sim 3 \times 10^{-5}$ AU, so the fractional difference in magnification as seen from the two locations would ordinarily be of this order. However, Holz & Wald (1996) pointed out that photon statistics alone do not necessarily prevent the detection of such a small effect. Moreover, the fractional difference is increased by approximately the magnification, so for EMEs the fractional difference in magnification between two observatories can be of order 1%. Because the magnification is so high, there is a high probability that the lens will transit (or nearly transit) the source which would permit measurement of the proper motion. Gould (1997) estimated that of order 30 EMEs occur per year toward the Galactic bulge, and that complete solutions could be obtained for a large fraction of them by follow-up observations. However, finding these EMEs would require a pixel-lensing (Gould 1996b) search of the entire bulge in real time. So far there are no plans to organize such a search.

Fourth, complete solutions can be obtained for some caustic-crossing binary events (Hardy & Walker 1995). The method is closely related to the EME method: caustic-crossing binary-lens events are automatically “extreme magnification events,” since the caustic has formally infinite magnification. Actually, the peak magnification is suppressed by the finite size of the source, just as it is when a point lens transits a finite source. The peak magnifications generally do not get as high as for EMEs because the magnification scales as the inverse square root of the distance from the caustic for binary-lens caustics and scales inversely for point lenses. Nevertheless, from the standpoint of measuring the parallax, what is important is not the magnification per se, but the logarithmic rate of change of the magnification with position in the Einstein ring. Basically, the magnification goes from peak to nearly zero as the lens crosses the radius of the source. If the angular radius of the source is small, then the logarithmic magnification gradient can be very large.

However, the size of the parallax effect does not depend only on the (known) distance between the observatories: it depends on the difference in the distance between the source and the caustic as seen from the two observatories. This difference is equal to the distance between the two observatories times the cosine of the angle between the normal to the caustic and the line connecting the observatories. Hence

one cannot measure the parallax using two observatories alone: one can only obtain a lower limit. By adding a third observatory (not collinear with the other two), this degeneracy can be broken and the parallax measured. Since it is always possible to measure the proper motion of a caustic-crossing binary, those events with parallax measurements can be solved completely.

Here we investigate this method more closely. In § 2 we identify the four quantities (in addition to the source distance) that must be measured to obtain a complete solution and give explicit formulae for the mass, distance, and speed of the lens in terms of these observables. In § 3 we present analytic expressions for the difference in magnification as seen from two observatories for the case when the source is close to but not yet crossing the caustic, and show that the measurable quantity is a degenerate combination of the parameter one would like to know and the secant of an unknown angle. In §§ 4 and 5, we show that this degeneracy can be broken either by making observations from a third noncollinear location or by observing two caustic crossings, each from two observatories. We indicate, however, that the latter is generally impractical. In § 6 we derive expressions for the magnification and its derivative during a caustic crossing. In § 7 we show that the quantity most directly determined from a caustic-crossing parallax measurement is \tilde{r}_* , essentially the physical size of the source projected through the lens onto the observer plane. We also show that all three of the other measurable quantities identified in § 2 depend on a correct modeling of the light curve as a whole, not just the caustic crossing. We derive analytic expressions for the signal-to-noise ratio (S/N) in terms of the geometry of the event and the characteristics of the source and telescopes in § 8. In particular, for sources above the sky the S/N depends only on the surface brightness of the source and not on its radius. In addition, $S/N \propto D^{-1}$, where $D \equiv D_{\text{os}} D_{\text{ol}} / D_{\text{ls}}$ and D_{os} , D_{ol} , and D_{ls} are the distances between the observer, source, and lens. These facts allow us to classify all possible events and to determine generally which have observable parallaxes.

We show that by using 1 m class telescopes it is possible to measure parallaxes for halo lenses seen toward the LMC and SMC, but it is not possible for LMC lenses and SMC lenses. One way to think about the difference is that the transverse velocity of a halo lens projected onto Earth is about 275 km s^{-1} , so it takes about 15 s to sweep from one continent to another 4000 km away. However, the projected velocity of an LMC lens is about 1500 km s^{-1} , so the lens moves across the ocean in 3 s. The magnification simply does not change enough in 3 s to detect the difference.

We find that robust measurements can be made for foreground disk lenses seen toward the Galactic bulge, but only marginal detections should be expected for bulge lenses. It should also be possible to measure the parallaxes of relatively nearby ($\lesssim 20 \text{ kpc}$) Galactic binary lenses detected toward M31, but not for those that are substantially more distant. Two groups are currently searching for lensing events toward M31 (Tomaney & Crofts 1996; Ansari et al. 1997). While no binaries (and indeed no confirmed lensing events) have yet been detected, these experiments are just now gearing up to become major efforts.

2. COMPLETE SOLUTIONS

In this paper we will show that it is possible, at least in

principle, to extract two parameters from a binary-lens microlensing event, t_* and \tilde{r}_* ,

$$t_* = \frac{D_{\text{ol}} \theta_*}{v}, \quad \tilde{r}_* = D \theta_*, \quad D \equiv \frac{D_{\text{ol}} D_{\text{os}}}{D_{\text{ls}}}. \quad (1)$$

Here θ_* is the angular size of the source, v is the transverse speed of the lens relative to the observer-source line of sight, and D_{ol} , D_{os} , and D_{ls} are the distances between the observer, lens, and source. There are three other observable quantities:

$$\theta_*, \quad t_E, \quad D_{\text{os}}. \quad (2)$$

The angular source size θ_* can be determined from its observed color and magnitude, and the estimated extinction can be determined using the relation $F_* = 10^{-0.4A} \pi \theta_*^2 S_*$. Here F_* is the observed flux, A is the extinction, and S_* is the surface brightness which is inferred from the dereddened color (i.e., from the temperature). The Einstein crossing time,

$$t_E = \frac{D_{\text{ol}} \theta_E}{v}, \quad \theta_E \equiv \left(\frac{4GM}{c^2 D} \right)^{1/2}, \quad (3)$$

can be determined from the overall fit to the light curve. Here θ_E is the angular Einstein radius, and M is the total mass of the binary lens. Finally, the distance D_{os} is approximately equal to the mean distance of the source population (e.g., the LMC). From these five parameters one can easily obtain the physically important quantities:

$$D_{\text{ol}} = \left(\theta_* + \frac{1}{D_{\text{os}}} \right)^{-1}, \quad (4)$$

$$D_{\text{ls}} = D_{\text{os}} \left(1 + \frac{\tilde{r}_*}{\theta_* D_{\text{os}}} \right)^{-1}, \quad (5)$$

$$v = \left(\frac{t_*}{\tilde{r}_*} + \frac{t_*}{D_{\text{os}} \theta_*} \right)^{-1}, \quad (6)$$

$$M = \frac{c^2}{4G} \theta_* \tilde{r}_* \left(\frac{t_E}{t_*} \right)^2. \quad (7)$$

Gould (1995b) has given an analogous set of equations in terms of the projected Einstein radius, $\tilde{r}_E \equiv D \theta_E$, the quantity measured by a parallax satellite or from Earth's motion during a long event. To make contact with these other forms of parallax measurement, we note that $\tilde{r}_* = (t_*/t_E) \tilde{r}_E$. The projected Einstein radius \tilde{r}_E is also useful for understanding the simplified picture of caustic-crossing parallaxes, which we present in the next two sections.

3. TWO OBSERVERS

Consider a microlensing event as observed from two different observatories. Let d_{12} be the projected separation between the observatories, i.e., the physical distance between them *projected onto the plane of the sky*. For a point source near a caustic, the magnification as seen by each observatory is given by

$$A_i^0 = \alpha (\Delta u_i)^{-1/2} + \gamma, \quad (8)$$

where Δu_i is the angular separation between the source and the caustic in units of θ_E as seen from each observatory ($i = 1, 2$), and α and γ are constants. In the range of interest,

$$d_{12} \ll \tilde{r}_E, \quad \tilde{r}_E \equiv D \theta_E, \quad (9)$$

the caustic can be approximated as a straight line. Hence

$$\Delta u_2 - \Delta u_1 = \frac{d_{12}}{\tilde{r}_E} \cos \theta_{12}, \quad (10)$$

where θ_{12} is the angle between the projected separation of the observatories and the normal to the caustic. The ratio of magnification is therefore given by

$$\begin{aligned} \frac{A_2^0}{A_1^0} &= \frac{\alpha[\Delta u_1 + (d_{12} \cos \theta_{12})/\tilde{r}_E]^{-1/2} + \gamma}{\alpha(\Delta u_1)^{-1/2} + \gamma} \\ &\approx 1 - \frac{1}{2} \frac{d_{12}}{\tilde{r}_E \Delta u_1} \cos \theta_{12}. \end{aligned} \quad (11)$$

4. THREE OBSERVERS

In equation (11) α , γ , and Δu_1 are all known from the overall fit to the light curve. In addition, d_{12} is known from terrestrial measurements. Hence the measurement of the flux ratio A_2^0/A_1^0 yields only the degenerate parameter combination $\tilde{r}_E \sec \theta_{12}$. To break this degeneracy, additional observations are required. In principle, two distinct types of additional observations could be used. First, one could observe the event from three observatories instead of two. The only requirement is that the three observatories should not be collinear, i.e., they must be vertices of a triangle. Now there are three unknowns (\tilde{r}_E , θ_{12} , and θ_{13}), but also three equations. These are (1) equation (11), (2) a similar equation for the flux ratio A_3^0/A_1^0 (which yields $\tilde{r}_E \sec \theta_{13}$), and (3) an additional expression that gives the relation between the angles,

$$\theta_{12} + \theta_{13} = \theta_{213}. \quad (12)$$

Here θ_{213} is the known angle between the line connecting observers 1 and 2 and the line connecting observers 1 and 3. Note that there is actually a sign ambiguity in equation (12) for the relation among the angles: it could also be $|\theta_{12} - \theta_{13}| = \theta_{213}$. However, this ambiguity is easily resolved by considering similar equations for θ_{132} and θ_{321} . Hence with three observers the degeneracy is broken, and \tilde{r}_E can be separately determined. Note that the light curves as seen from the three observatories will look essentially identical, except that they are displaced in time.

As pointed out to us by the referee, it is possible at least in principle to break the degeneracy by observations from two observatories that are moving relative to one another. This is related to the fact that it is possible to break the degeneracy in the satellite parallax problem (Gould 1994b) by making use of the Earth-satellite relative motion (Gould 1995a), although in that case the degeneracy is discrete, whereas here it is continuous. As a practical matter, the moving observer can go no faster than a low-Earth orbital velocity, $v_{\text{orb}} = 8 \text{ km s}^{-1}$. There are two limiting cases to consider. First, suppose that $t_* \gtrsim P_{\text{orb}} = 2\pi r_{\oplus}/v_{\text{orb}} \sim 90$ minutes. The satellite then will effectively observe the event from two widely separated locations, and (assuming that the effect is large enough to be measured from two separated observatories in the first place) the degeneracy will easily be broken. However, $t_* \gtrsim P_{\text{orb}}$ implies $r_* \gtrsim 2\pi(D_{\text{os}}/D_{\text{ol}})(v/v_{\text{orb}})r_{\oplus} \sim 9r_{\oplus}$ for typical parameters ($D_{\text{os}}/D_{\text{ol}} \sim 5$, $v \sim 220 \text{ km s}^{-1}$). As discussed in § 8, red giants are not very suitable source stars, so this option requires bright main sequence stars. In the other limit, $t_* \lesssim P_{\text{orb}}$, the principal effect will be that the light curve as seen by the

moving observer will change at a rate that differs fractionally by $v_{\text{orb}}/\tilde{v} \sim 3\%$ from the rate seen by the stationary observer. If this can be detected, then the degeneracy can be broken. Actually, it is only the component of v_{orb} perpendicular to the line of sight separating the observatories that contributes to breaking the degeneracy: the rate change due to motion in the parallel direction can be predicted from the difference in the two light curves and does not provide independent information. To detect a 3% difference in the caustic crossing time will be substantially more difficult than detection of an $\sim 15\%$ flux difference (see § 8) between two observatories, but it may be possible.

5. TWO CAUSTIC CROSSINGS

In principle, it is possible to break the degeneracy even in the case of two stationary observers, provided that both observers monitor *two* caustic crossings, *a* and *b*. One can then measure $\tilde{r}_E \sec \theta_{12a}$ and $\tilde{r}_E \sec \theta_{12b}$. So far, there are two equations, but three unknowns (\tilde{r}_E , θ_{12a} , and θ_{12b}). A third equation can be obtained as follows: the angles at which the lens crosses the caustics, ϕ_a and ϕ_b , are known from the binary-lens solution for the overall light curve. The difference between these angles is equal to the angle between the two tangents to the caustics at the crossing points. The difference between θ_{12a} and θ_{12b} (the angles between the line connecting the observatories and the normals to the caustics) is likewise equal to the angle between the tangents to the caustics. Hence

$$\theta_{12a} - \theta_{12b} = \phi_a - \phi_b. \quad (13)$$

However, as a practical matter there is little opportunity to make such measurements because there is no warning of the first caustic, and hence there will not be enough time to prepare for the measurements.

6. THE MAGNIFICATION NEAR THE CAUSTIC

The previous results concern the case where the source is small relative to its separation from the caustic. Let us now consider the case where the source size and separation are comparable. The magnification of a point source continues to be given by equation (8), but for a finite source we must integrate over the surface brightness of the source,

$$A = \frac{\int_0^r \rho d\rho \int_0^{2\pi} d\psi A^0[\Delta u(\rho, \psi)] J(\rho, \psi)}{\int_0^r \rho d\rho \int_0^{2\pi} d\psi J(\rho, \psi)}, \quad (14)$$

where ρ and ψ are polar coordinates, $J(\rho, \psi)$ is the intensity as a function of polar position, $\Delta u(\rho, \psi) = \Delta u^0 + (\rho/\theta_E) \cos \psi$, and Δu^0 is the separation of the center of the star from the caustic. For simplicity, we assume uniform surface brightness and find

$$A(\eta) = \alpha \left(\frac{\theta_E}{\theta_*} \right)^{1/2} G(\eta) + \gamma, \quad (15)$$

where

$$G(\eta) = \frac{2}{\pi} \int_{\max(\eta, -1)}^1 \left(\frac{1-x^2}{x-\eta} \right)^{1/2} dx. \quad (16)$$

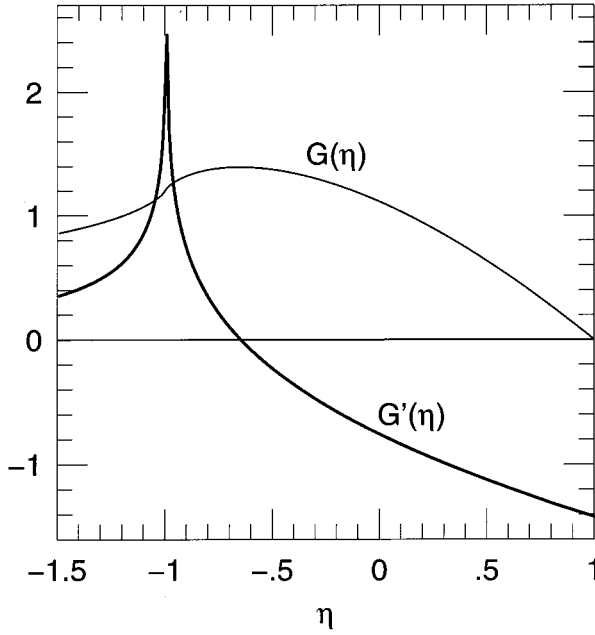


FIG. 1.—Normalized light-curve profile $G(\eta)$ (solid line) for a uniform source crossing a binary-lens caustic. The magnification is given by $\alpha(\theta_E/\theta_*)^{1/2}G(\eta) + \gamma$, where α and γ are constants, θ_E is the angular Einstein radius, θ_* is the angular radius of the source, and η is the separation of the source center from the caustic in units of θ_* . The sign convention is such that $\eta > 0$ if the source is outside the caustic. The parallax effect (difference in magnifications as seen from two observatories) is $\alpha(\theta_E/\theta_*)^{1/2}G'(\eta)\Delta\eta$, where G' (heavy line) is the derivative of G , $\Delta\eta = d_{12} \cos \theta_{12}/\tilde{r}_*$ is the difference in values of η between the two observatories, d_{12} is the distance between the observatories projected on the plane of the sky, θ_{12} is the angle between the normal to the caustic and the line connecting the observatories, $\tilde{r}_* = D\theta_*$ is the source radius projected onto the plane of the observer, $D = D_{os}D_{ls}/D_{ol}$, and D_{ol} , D_{ls} , and D_{os} are the distances between the observer, lens, and source. Near the end of the caustic crossing ($\eta = 1$), the figure shows that $|G'(\eta)| \rightarrow 2^{1/2}$, while $G(\eta) \rightarrow 0$, so the fractional difference in magnification reaches a maximum.

Here η is the dimensionless separation between the source and the caustic, given by

$$\eta = \frac{\Delta u^0 \theta_E}{\theta_*} \quad (\eta < 1). \quad (17)$$

The derivative of G is given by

$$G'(\eta) = \frac{2}{\pi} \int_{\max(\eta, -1)}^1 \frac{x}{[(x - \eta)(1 - x^2)]^{1/2}} dx. \quad (18)$$

The functions $G(\eta)$ and $G'(\eta)$ are shown in Figure 1.

7. SUMMARY OF MEASURABLE QUANTITIES

Before continuing, we pause to assess how the parameters t_* , t_E , \tilde{r}_* , and θ_* depend on the observations, and to what degree their estimation depends on the model of the binary lens that is derived from the full light curve. Both t_* and t_E can be derived from observations from a single observatory, and both depend critically on a correct modeling of the binary lens. The shape of the light curve during a caustic crossing, $G(\eta)$, is shown in Figure 1. The caustic crossing time Δt is defined as the time necessary to move $\Delta\eta = 1$ in Figure 1, and is therefore quite robustly measured from the caustic-crossing data. However, $t_* \equiv \Delta t \sin \phi$, where ϕ is the angle between the velocity of the lens relative to the

source and the caustic. The determination of this angle depends on the overall light curve, and good data over large parts of the light curve are necessary for an accurate measurement (see, e.g., Albrow et al. 1999a). The determination of t_E also depends on the overall light curve (see, e.g., Albrow et al. 1999a).

By contrast, the determination of \tilde{r}_* does not depend on the global light curve, but only on the caustic crossing. The measured quantity

$$\frac{A(\eta_2)}{A(\eta_1)} = \frac{G(\eta_2) + (\gamma/\alpha)(\theta_*/\theta_E)^{1/2}}{G(\eta_1) + (\gamma/\alpha)(\theta_*/\theta_E)^{1/2}} \approx 1 + \frac{G'(\eta)}{G(\eta)} \Delta\eta, \quad (19)$$

where

$$\Delta\eta \equiv \eta_2 - \eta_1 = \frac{d_{12} \cos \theta_{12}}{\tilde{r}_*} \quad (20)$$

depends only weakly on the binary-lens model parameters α and γ . Since $G'(\eta)/G(\eta)$ is well determined from the caustic-crossing data, and d_{12} is known from terrestrial measurements, the degenerate combination $\tilde{r}_* \sec \theta_{12}$ is well determined from the caustic-crossing measurements from two sites. As discussed in § 4, this degeneracy can be broken by observations from the third site. Finally, the color (say, $V - I$) of the source can be determined from the approach to the second caustic crossing because the magnification is so high that blending plays very little role, and the star is not yet resolved by the caustic so the magnified source has very nearly the same color as the intrinsic source. On the other hand, determination of the intrinsic magnitude of the source is dependent on correct modeling of the decomposition of the observed flux into (magnified) source and blend. The angular radius of the star, θ_* , depends not only on the color and magnitude of the source, but also on the reddening; however, this dependence is relatively weak (see, e.g., Albrow et al. 1999a).

8. FEASIBILITY OF MAKING THE MEASUREMENT

How practical is the measurement of \tilde{r}_* ? There are two general requirements for the observations. First, the relative difference of the two magnifications should be great enough to be recognized as a valid result. We write this requirement in the form

$$\frac{|A(\eta_2) - A(\eta_1)|}{A(\eta_1)} > p, \quad (21)$$

where we suggest $p \sim 0.01$. Since $A(\eta_2) - A(\eta_1) \cong \alpha(\theta_E/\theta_*)^{1/2}G'(\eta)(\eta_2 - \eta_1)$, equation (21) can be conveniently rewritten

$$\frac{|G'(\eta)\Delta\eta|}{G(\eta) + Z} > p, \quad Z = \frac{\gamma}{\alpha} \left(\frac{\theta_*}{\theta_E} \right)^{1/2}, \quad (22)$$

where $\Delta\eta \equiv \eta_2 - \eta_1$. The left-hand side of equation (22) reaches a maximum at $\eta = 1$ (see Fig. 1). (Formally the maximum is at $\eta = -1$, but in practice this peak is too short to be resolved.) One may analytically show that $G'(1) = -2^{1/2}$ and $G(1) = 0$. Therefore, equation (22) may be rewritten

$$\frac{|\Delta\eta|}{Z} = 2^{1/2} \frac{d_{12} |\cos \theta_{12}| / \tilde{r}_*}{(\gamma/\alpha)(\theta_*/\theta_E)^{1/2}} > p, \quad (23)$$

or

$$0.15\alpha\left(\frac{\gamma}{5}\right)^{-1}\left(\frac{d_{12}}{3000\text{ km}}\right)\left(\frac{D_{\text{os}}\theta_E}{20\text{ AU}}\right)^{1/2}\left(\frac{r_*}{1.5r_\odot}\right)^{-3/2} \\ \times \left(\frac{|\cos\theta_{12}|}{0.7}\right)\left(\frac{D_{\text{ls}}/D_{\text{ol}}}{5}\right) > p. \quad (24)$$

Hence for halo lenses seen toward the LMC (for which the above normalizations of the parameters are “typical”), the condition is relatively easily met. However, for LMC lenses ($D_{\text{ls}}/D_{\text{ol}} \sim 0.1$, $D_{\text{os}}\theta_E \sim 3\text{ AU}$), the left-hand side is smaller by a factor of ~ 130 , so parallax measurements would be extremely difficult.

Note that during the period when the magnification difference is near maximum ($\eta \lesssim 1$), the *sign* of the difference remains constant, so it is possible to detect the difference even when the exposure times are longer than the $\sim 15\text{ s}$ difference in crossing time. Of course, if the exposure times are a significant ($\gtrsim 10\%$) fraction of the crossing time ($t_* \sim 30\text{ minutes}$), then one will start to lose resolution of the light curve. Hence exposure times of order 1 minute are indicated.

The second requirement is that the S/N be sufficiently high for robust detection. For an exposure time t_{exp} , the S/N is given by

$$\frac{S}{N} = \frac{\alpha(t_E/t_*)^{1/2} |G'(\eta)\Delta\eta| F_* \Gamma t_{\text{exp}}}{(2\{\alpha(t_E/t_*)^{1/2} G(\eta) + \gamma\} F_* + F_{\text{sk}}) \Gamma t_{\text{exp}}^{1/2}}, \quad (25)$$

where F_* is the unmagnified flux of the source, Γ is the rate of photon detection per unit flux, and F_{sk} is the flux from the sky within the aperture of the point spread function (PSF). For a typical observing system and a $V_* = 20$ source, $F_* \Gamma = 25\text{ s}^{-1}(\mathcal{D}/\text{m})^2$, where \mathcal{D} is the diameter of the mirror. Note that we have made use of the fact that $t_E/t_* = \theta_E/\theta_*$. We adopt F_{sk} equivalent to $V_{\text{sk}} = 20$, which corresponds to a sky brightness of $V = 21.3\text{ mag arcsec}^{-2}$ and a $1''$ PSF.

Equation (25) can be rewritten

$$\frac{S}{N} = \frac{\alpha |G'(\eta)|}{[\alpha(\theta_E/\theta_*)^{1/2} G(\eta) + B]^{1/2}} \left[\frac{F_* \Gamma t_{\text{exp}}}{2} \left(\frac{t_E}{t_*} \right) \right]^{1/2} \\ \times \frac{d_{12} |\cos\theta_{12}|}{\tilde{r}_*}, \quad (26)$$

where

$$B = \gamma + \frac{F_{\text{sk}}}{F_*}. \quad (27)$$

Again, inspection of Figure 1 shows that the S/N will be maximized near the end of the caustic crossing, $\eta = 1$, where $G'(\eta) = -2^{1/2}$ and $G(\eta) = 0$. In this limit, the first term of equation (26) approaches $(2/B)^{1/2}$. Hence there are two limits, depending on whether the magnified source *outside the caustic* is brighter or fainter than the sky integrated over the PSF aperture.

If the sky dominates, the S/N is given by

$$\frac{S}{N} = 13\alpha 10^{(V_{\text{sk}} - V_*)/5} \left(\frac{S_*}{S_\odot} \right)^{1/2} \left(\frac{D_{\text{os}}}{50\text{ kpc}} \right)^{-1} \left(\frac{t_E}{40\text{ day}} \right)^{1/2} \\ \times \left(\frac{D_{\text{ls}}/D_{\text{ol}}}{5} \right) \left(\frac{t_{\text{exp}}}{t_*/10} \right)^{1/2} \left(\frac{d_{12}}{3000\text{ km}} \right) \left(\frac{|\cos\theta_{12}|}{0.7} \right) \left(\frac{\mathcal{D}}{1\text{ m}} \right), \quad (28)$$

where S_* is the (reddened) surface brightness of the star, S_\odot is the surface brightness of the Sun, and where we have made use of the fact that $F_*/r_*^2 = S_*/D_{\text{os}}^2$. If the source dominates, then

$$\frac{S}{N} = 6\alpha \left(\frac{\gamma}{5} \right)^{-1/2} \left(\frac{S_*}{S_\odot} \right)^{1/2} \left(\frac{D_{\text{os}}}{50\text{ kpc}} \right)^{-1} \left(\frac{t_E}{40\text{ day}} \right)^{1/2} \left(\frac{D_{\text{ls}}/D_{\text{ol}}}{5} \right) \\ \times \left(\frac{t_{\text{exp}}}{t_*/10} \right)^{1/2} \left(\frac{d_{12}}{3000\text{ km}} \right) \left(\frac{|\cos\theta_{12}|}{0.7} \right) \left(\frac{\mathcal{D}}{1\text{ m}} \right). \quad (29)$$

For simplicity, we have focused on the S/N achieved during the last $0.1t_*$ of the caustic crossing. The total S/N could be improved by a factor of ~ 2 by monitoring the entire crossing which, for a halo binary lens toward the LMC, lasts about $3t_* \sim 30\text{ minutes}$. These results indicate that for halo binary lenses, parallax is measurable only for sources down to about 2 mag fainter than the sky ($V_* \lesssim 22$), and then only for sources hotter than the Sun. Even for sources that are brighter than the sky (when magnified just outside the caustic), the source should be bluer than the Sun to obtain a good measurement of the parallax. In particular, red giant sources (no matter how bright) will have relatively low S/N . Equations (28) and (29) show that it is not possible (with modest-size telescopes) to measure parallaxes for LMC binary lenses (for which $D_{\text{ls}}/D_{\text{ol}} \lesssim 0.1$).

To consider other lines of sight, first note that the factors $(D_{\text{os}}/50\text{ kpc})^{-1}(D_{\text{ls}}/D_{\text{ol}}/5)$ can be written more simply as $(D/10\text{ kpc})^{-1}$. See equation (1).

For bulge events, the optimal sources are turnoff stars because they are the bluest common bulge stars. Taking account of extinction ($A_V \sim 1.5$), we evaluate the parameter combination in equations (28) and (29) as $(S_*/S_\odot)(D/10\text{ kpc})^{-1} \sim D_{\text{ls}}/D_{\text{os}}$. Thus parallaxes could be easily measured for foreground disk binary lenses ($D_{\text{ol}} \gtrsim D_{\text{ls}}$), and with some difficulty for bulge binaries ($D_{\text{ls}}/D_{\text{ol}} \sim 1/3$).

Parallaxes might also be measured for Galactic halo binary lenses observed toward M31. In this case $(D/10\text{ kpc})^{-1} \simeq (D_{\text{ol}}/10\text{ kpc})^{-1}$. Thus, especially for blue main-sequence sources (with their high surface brightness), reasonable S/N could be obtained out to $D_{\text{ol}} \sim 20\text{ kpc}$.

The requirement that the observations be done at night (which is usually taken for granted) imposes considerable additional constraints in the present case. First, parallax measurements are generally possible only during the autumn and winter because widely separated observatories are not usually in darkness at the same time near the summer solstice. For the LMC and SMC in particular, the observations must be done in autumn and winter because the only suitable (noncollinear) location for a third observatory is Antarctica, which is in daytime during the entire spring and summer. Note that for the LMC this is particularly awkward since it is under the pole in winter. Autumn and winter are, of course, the most favorable times to observe the bulge. Moreover, there are numerous northern observatories around the globe that can view the bulge, at least for brief periods, and which could therefore serve as third noncollinear observatories. M31 observations are most feasible in October when it is up all night and when the nights are reasonably long. Since most northern observatories are at $\sim 30^\circ$ latitude, it will be somewhat difficult to obtain a long north-south baseline for the third noncollinear observatory.

Still another requirement is that the different observatories be “on the same system,” so that the observations can be compared rigorously. The most critical aspect is that the time of observation be recorded accurately to the nearest second (since the difference in caustic crossing times is only of order 15 s). While this poses no difficulty in principle, it should not be taken for granted: many control systems query the clock when the shutter opens, but do not enforce a prompt response, so that the recorded time could differ from the clock time by many seconds. In addition, the system clock may not be directly tied to an international standard. Thus care is required to assure accurate time comparisons. The second aspect is that the different observatories should be on the same photometric system. Actually, this is not a strict requirement provided the different observatories both observe the end of the caustic cross-

ing. Then, in effect, what is compared is simply the time of crossing, and it does not matter if the two sets of observations are even in the same bands. However, a common system becomes more critical if one of the observatories misses the crossing so that one is only comparing different magnifications at the same time. Fortunately, the experience of the PLANET collaboration (Albrow et al. 1998, 1999b) is that by using common secondary standards, it is possible to align photometry at different observatories to substantially better than 1%, i.e., much better than the requirement of equation (24).

We thank Scott Gaudi for his careful comments on the manuscript, and the referee, Robert Nemiroff, for making a number of useful suggestions and comments. This work was supported in part by grant AST 97-27520 from the NSF.

REFERENCES

- Afonso, C., et al. 1998, *A&A*, 337, L17
 Albrow, M., et al. 1998, *ApJ*, 509, 687
 ———. 1999a, *ApJ*, 512, 672
 ———. 1999b, *ApJ*, submitted
 Alcock, C., et al. 1995, *ApJ*, 454, L125
 ———. 1997a, *ApJ*, 479, 119
 ———. 1997b, *ApJ*, 486, 697
 ———. 1997c, *ApJ*, 491, 436
 ———. 1999, *ApJ*, submitted
 Ansari, R., et al. 1997, *A&A*, 324, 843
 Aubourg, E., et al. 1993, *Nature*, 365, 623
 Bennett, D., et al. 1997, *BAAS*, 191, 8308
 Boden, A. F., Shao, M., & Van Buren, D. 1998, *ApJ*, 502, 538
 Boutuex, T., & Gould, A. 1996, *ApJ*, 462, 705
 Buchalter, A., & Kamionkowski, M. 1997, 482, 782
 Evans, N. W., Gyuk, G., Turner, M. S., & Binney, J. J. 1998, *ApJ*, 501, L45
 Gaudi, B. S., & Gould, A. 1997, *ApJ*, 477, 152
 Gould, A. 1992, *ApJ*, 392, 442
 ———. 1994a, *ApJ*, 421, L71
 ———. 1994b, *ApJ*, 421, L75
 ———. 1995a, *ApJ*, 441, L21
 ———. 1995b, *ApJ*, 447, 491
 ———. 1996a, *PASP*, 108, 465
 ———. 1996b, *ApJ*, 470, 201
 ———. 1997, *ApJ*, 480, 188
 ———. 1998, *ApJ*, 506, 253
 Gould, A. 1999, *ApJ*, 514, 869
 Gould, A., Miralda-Escudé, J., & Bahcall, J. N. 1994, *ApJ*, 423, L105
 Gould, A., & Salim, S. 1999, *ApJ*, submitted
 Han, C., & Gould, A. 1996, *ApJ*, 467, 540
 ———. 1997, *ApJ*, 480, 196
 Hardy, S. J., & Walker, M. A. 1995, *MNRAS*, 276, L79
 Holz, D. E., & Wald, R. M. 1996, *ApJ*, 471, 64
 Maoz, D., & Gould, A. 1994, *ApJ*, 425, L67
 Marković, D. 1998, *MNRAS*, 299, 929
 Nemiroff, R. J., & Wickramasinghe, W. A. D. T. 1994, *ApJ*, 424, L21
 Palanque-Delabrouille, N., et al. 1998, *A&A*, 332, 1
 Refsdal, S. 1966, *MNRAS*, 134, 315
 Rhie, S. H., Becker, A. C., Bennett, D. P., Fragile, P. C., Johnson, B. R., King, L. J., Peterson, B. A., & Quinn, J. 1999, *ApJ*, submitted
 Sahu, K. C. 1994, *Nature*, 370, 275
 Tomaney, A., & Crotts, A. P. S. 1996, *AJ*, 112, 2872
 Udalski, A., et al. 1994, *Acta Astron.*, 44, 165
 Udalski, A., Szymański, M., Kubiak, M., Pietrzyński, G., Woźniak, P., & Żebruń, K. 1998, *Acta Astron.*, 48, 431
 Witt, H. J., & Mao, S. 1994, *ApJ*, 429, 66
 Wu, X.-P. 1994, *ApJ*, 435, 66
 Zaritsky, D., & Lin, D. N. C. 1997, *AJ*, 114, 2545
 Zhao, H. 1998, *MNRAS*, 294, 139
 Zhao, H., Rich, R. M., & Spergel, D. N. 1996, *MNRAS*, 282, 175
 Zhao, H., Spergel, D. N., & Rich, R. M. 1995, *ApJ*, 440, L13